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infinitesimal of higher order than the first; that is, $PA + PB$ is constant up to and including the first order. Hence (as shown by the laws of reflection for light, or from the properties of the ellipse), PA and PB must make equal angles with PC. By symmetry, the angles subtended at P by the three sides of the triangle ABC must all be 120°.

It is now easy to construct the point P. Draw upon two sides of the triangle arcs of 120°. Their intersection determines P, and the arc of 120° on the third side of the triangle passes through P. (The construction breaks down when one angle of the triangle is greater than 120°, and in this case the vertex of that angle is the solution of the problem.)

I should be glad to see other analytic solutions of the problem which are as simple. See the solution by Dunham Jackson in the January 1917 MONTHLY, which appeared after this note was sent to the editor.

II. RELATING TO A PROBLEM IN MINIMA DISCUSSED BY PROFESSOR DUNHAM JACKSON IN THE JANUARY, 1917, NUMBER (P. 46) OF THIS MONTHLY.

BY ROGER A. JOHNSON, Western Reserve University, Cleveland, Ohio.

The problem is to determine a point the sum of whose distances from the vertices of a given triangle shall be a minimum.

Professor Jackson has apparently overlooked the fact that this problem admits an elementary geometric solution due to Steiner (*Collected Works*, Vol. II, p. 729). The following is a development of this solution; for more detailed treatment and historical notes, see Emmerich, *Die Brocard'sche Gebilde*, § 42.

Lemma I. The sum of the distances to the sides of an equilateral triangle, from a point inside the triangle, is constant and equal to the altitude. Moreover, if from any point the perpendiculars to the sides of the equilateral triangle make angles of 120° (and not 60°) with one another, the point is inside the triangle.

Lemma II. Let ABC be a triangle, each angle of which is less than 120°. There is a point P, and only one, such that $\angle BPC = \angle CPA = \angle APB = 120°$.

Lemma III. Through A, B, C, draw MN, NL, LM, perpendicular to PA, PB, PC, respectively. Then LMN is an equilateral triangle and P is inside it.

THEOREM. *If Q is any point of the plane, other than P, then*

$$QA + QB + QC > PA + PB + PC.$$

For $QA + QB + QC$ is greater than the sum of the perpendiculars from Q to MN, NL, LM; and the sum of these perpendiculars is greater than or equal to $PA + PB + PC$, according as Q lies outside or inside triangle LMN.

The point P is one of two points called *isogonic centers*, which have numerous interesting properties (cf. Emmerich, *l. c.*). If any angle of the triangle is greater than 120°, it falls outside the triangle and therefore fails to yield a minimum. As Professor Jackson indicates, it is not hard in this case to show that the vertex of the obtuse angle is the desired point. To do this by the same method we have just used, we assume that angle B is greater than 120°. Through A and C, respectively, let XAY and XBZ be drawn perpendicular to BA and BC respectively; through B draw a line YZ so that $\triangle XYZ$ is isosceles, and $\angle YXZ$ is less than 60°; then $ZY < XY$.

Let PF, PG, PH, be perpendicular to XY, YZ, ZX, respectively. Then

$$XY \cdot PF + XZ \cdot PH + YZ \cdot PG = XY \cdot BA + XZ \cdot BC.$$

Let $m = XY = XZ$. Then, since $YZ \cdot PG < XY \cdot PG$,

$$m(PF + PG + PH) > m(BA + BC),$$

and

$$PF + PG + PH > BA + BC.$$

Also, as before, $PA + PB + PC \geqq PF + PG + PH$, which establishes the result.

III. Relating to the Exponential Function.

By Otto Dunkel, Washington University.

An interesting treatment of the elementary transcendental functions was given by A. Hurwitz in the *Mathematische Annalen*, Vol. 70, 1911, entitled "Über die Einführung der elementaren transzendenten Funktionen in der algebraischen Analysis." The method used by Hurwitz was somewhat similar to the one used by Professor Huntington in his article "An Elementary Theory of the Exponential and Logarithmic Functions" in the September, 1916, number of the Monthly, pp. 241–246, except that Hurwitz discussed first the function $\log x$ and then derived sequences for the definition of e^x of the type used by Huntington. About the time of the appearance of Hurwitz's article the writer developed a treatment of the exponential function similar to the one given by Huntington but somewhat simpler in the fact that the inequality $(1 + d)^m > 1 + md$ was not required. This makes the proof more elementary, as the proof of this inequality in Huntington's article is made to depend partly upon the binomial theorem and partly upon an additional proof for a remaining case. This advantage is gained by using sequences in which each exponent is double the preceding, for then it is a very easy matter to prove the increasing and decreasing character of the two sequences, and having done this the sequences themselves supply the place of the above mentioned inequality in the subsequent reasonings. A brief sketch of how this can be effected is given below.

Given the sequences

$$(A) \quad 1 + x, \left(1 + \frac{x}{2}\right)^2, \left(1 + \frac{x}{4}\right)^4, \left(1 + \frac{x}{8}\right)^8, \cdots, \left(1 + \frac{x}{m}\right)^m, \left(1 + \frac{x}{2m}\right)^{2m}, \cdots$$

$$(B) \quad (1 - x)^{-1}, \left(1 - \frac{x}{2}\right)^{-2}, \left(1 - \frac{x}{4}\right)^{-4}, \left(1 - \frac{x}{8}\right)^{-8}, \cdots, \left(1 - \frac{x}{m}\right)^{-m}, \left(1 - \frac{x}{2m}\right)^{-2m}, \cdots,$$